Here is a revised version of my paper entitled " **Stability of the Vacuum Non Singular Black Hole**" after taking into account your comments. I have made a slight modifications in the "**Introduction**", added the first paragraph and modified the second one. Also I have added three references related to these modifications. I would like to publish this paper as an article in your magazine "Chaos Solitons and Fractals".

Please send any comments by e.mail to "nashed@asunet.shams.eun.eg" or by ordinary mail to:

Gamal. G.L. Nashed
Ain Shams University,
Faculty of Science,
Mathematics Department,
Abbassia, Cairo,
Egypt.

Finally please Sir, accept all my best regards

Sincerely Your

Gamal G.L. Nashed

Stability of the Vacuum Non Singular Black Hole

Gamal G.L. Nashed

Mathematics Department, Faculty of Science, Ain Shams University, Cairo, Egypt

e-mail:nashed@asunet.shams.edu.eg

The singularity of the black hole solutions obtained before in Møller's theory are studied. It is found that although the two solutions reproduce the same associated metric the asymptotic behavior of the scalars of torsion tensor and basic vector are quite different. The stability of the associated metric of those solutions which is spherically symmetric non singular black hole is studied using the equations of geodesic deviation. The condition for the stability is obtained. From this condition the stability of the Schwarzschild solution and di Sitter solution can be obtained.

1. Introduction

A black holes is a singularity in Einstein field equations. This can be happen when a gravitational collapse takes place and continue until the surface of the star approaches the Schwarzschild radius, i.e., $r = 2m$ [1]. Hawking and collaborators discovered that the laws of thermodynamics have an exact analogues in the properties of black holes [1, 2, 3, 4]. As a black hole emits particles, its mass and size steadily decrease. This makes it easier to tunnel out and so the emission will continue at an ever-increasing rate until eventually the black hole radiates itself out of existence. In the long run, every black hole in the universe will evaporates in this way. Sidharth [5] shown that a particle can be treated as a relativistic vortex, that is a vortex where the velocity of a circulation equals that of light or a spherical shell, whose constituents are again rotating with the velocity of light or as a black hole described by the Kerr-Newman metric for a spin $\frac{1}{2}$ particles.

Dymnikova [6] derived a static spherically symmetric black hole solution in orthodox general relativity assuming a specific form of the stress-energy momentum tensor. This solution practically coincides with the Schwarzschild solution for large r, for small r it behaves like the de Sitter solution and describes a spherically symmetric black hole singularity free everywhere [6]. It is shown that the metric of this solution has two event horizons, one related to the external horizon and the other is the internal horizon which is the Cauchy horizon [6]. It is shown that both of the horizons are removable. Dymnikova [6] has shown that this solution is regular at $r = 0$ and so it is nonsingular everywhere, so it is called a "nonsingular black hole". It has been proved that it is possible to treat this specific form of the stress-energy momentum tensor as corresponding to an r-dependent cosmological term $\Lambda_{\mu\nu}$, varying from $\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}$ as $r \to 0$ to $\Lambda_{\mu\nu} = 0$ as $r \to \infty$ [7]. More recently [8], the spherically symmetric nonsingular black hole has been used to prove that a baby universe inside a Λ black hole can be obtained in the case of an eternal black hole. Also it has been shown that the probability of a quantum birth of a baby universe can not be neglected due to the existence of an infinite number of Λ white hole structures.

In an earlier paper [9] the author used a spherically symmetric tetrad constructed by Robertson [10] to derive two different spherically symmetric vacuum nonsingular black hole solutions of Møller's field equations assuming the same form of the vacuum stress-energy momentum tensor given in [6]. He also calculated the energy content of these solutions.

It is the aim of the present paper to study the singularity of the two solutions obtained before [9] and then derive the condition for the stability of these solutions using the geodesic deviation [11]. In section 2, a brief review of the two solutions obtained before are given. The singularity problem of these solutions are studied in section 3. The condition of the stability for the vacuum non singular black hole solution is given in section 4. Section 5 is devoted to main results.

Computer algebra system Maple 6 is used in some calculations.

2. Spherically symmetric nonsingular black hole solutions

Dymnikova [6] in 1992 has obtained a static spherically symmetric nonsingular black hole solution in general relativity which is expressed by

$$ds^2 = (1 - \frac{R_g(r)}{r})dt^2 - \frac{dr^2}{(1 - \frac{R_g(r)}{r})} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2, \tag{1}$$

where

$$R_g(r) = r_g(1 - e^{-r^3/{r_1}^3}), \qquad {r_1}^3 = {r_0}^2 r_g, \qquad {r_0}^2 = \frac{3}{8\pi\epsilon_0}, \qquad and \qquad r_g = 2M. \tag{2}$$

This black hole is regular at r=0 as we will see from the study of the singularity problem. For the Einstein field equation to be satisfied the stress-energy momentum tensor must take the form [6]

$$\begin{aligned} {T_0}^0 &= {T_1}^1 = \epsilon_0 e^{-r^3/{r_1}^3}, \\ {T_2}^2 &= {T_3}^3 = \epsilon_0 e^{-r^3/{r_1}^3}\left(1 - \frac{3r^3}{2{r_1}^3}\right). \end{aligned} \tag{3}$$

In a previous paper the author used the teleparallel spacetime in which the fundamental fields of gravitation are the parallel vector fields ${b_k}^\mu$. The component of the metric tensor $g_{\mu\nu}$ are related to the dual components ${b^k}_\mu$ of the parallel vector fields by the relation

$$g_{\mu\nu} = \eta_{ij} {b^i}_\mu {b^j}_\nu, \tag{4}$$

where $\eta_{ij} = diag.(-,+,+,+)$. The nonsymmetric connection ${\Gamma^\lambda}_{\mu\nu}{}^*$ are defined by

$${\Gamma^\lambda}_{\mu\nu} = {b_k}^\lambda {b^k}_{\mu,\nu}, \tag{5}$$

as a result of the absolute parallelism [12].

The gravitational Lagrangian L of this theory is an invariant constructed from the quadratic terms of the torsion tensor

$${T^\lambda}_{\mu\nu} \stackrel{\text{def.}}{=} {\Gamma^\lambda}_{\mu\nu} - {\Gamma^\lambda}_{\nu\mu}. \tag{6}$$

The gravitational Lagrangian is given by [13]

$$\mathcal{L} \stackrel{\text{def.}}{=} -\frac{1}{3\kappa}\left(t^{\mu\nu\lambda}t_{\mu\nu\lambda} - v^\mu v_\mu\right) + \xi a^\mu a_\mu. \tag{7}$$

*Latin indices $(i, j, k, \cdots)$ designate the vector number, which runs from (0) to (3), while Greek indices $(\mu, \nu, \rho, \cdots)$ designate the world-vector components running from 0 to 3. The spatial part of Latin indices is denoted by $(a, b, c, \cdots)$, while that of Greek indices by $(\alpha, \beta, \gamma, \cdots)$.

Here ξ is a constant parameter, κ is the Einstein gravitational constant and $t_{\mu\nu\lambda}, v_\mu$ and a_μ are the irreducible components of the torsion tensor:

$$
\begin{aligned}
t_{\lambda\mu\nu} &= \frac{1}{2}\left(T_{\lambda\mu\nu}+T_{\mu\lambda\nu}\right)+\frac{1}{6}\left(g_{\nu\lambda}V_\mu+g_{\mu\nu}V_\lambda\right)-\frac{1}{3}g_{\lambda\mu}V_\nu,\\
V_\mu &= {T^\lambda}_{\lambda\nu},\\
a_\mu &= \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma},
\end{aligned}
\tag{8}
$$

where $\epsilon_{\mu\nu\rho\sigma}$ is defined by

$$
\epsilon_{\mu\nu\rho\sigma} \stackrel{\text{def.}}{=} \sqrt{-g}\delta_{\mu\nu\rho\sigma} \tag{9}
$$

with $\delta_{\mu\nu\rho\sigma}$ being completely antisymmetric and normalized as $\delta_{0123}=-1$.

By applying the variational principle to the Lagrangian (7), the gravitational field equation are given by [13][†]:

$$
G_{\mu\nu}+K_{\mu\nu} = -\kappa T_{(\mu\nu)}, \tag{10}
$$

$$
{b^i}_\mu {b^j}_\nu \partial_\lambda(\sqrt{-g}{J_{ij}}^\lambda) = \lambda\sqrt{-g}T_{[\mu\nu]}, \tag{11}
$$

where the Einstein tensor $G_{\mu\nu}$ is defined by

$$
G_{\mu\nu} = R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R, \tag{12}
$$

$$
{R^\rho}_{\sigma\mu\nu} = \partial_\nu \left\{{\rho \atop \sigma\mu}\right\} - \partial_\mu \left\{{\rho \atop \sigma\nu}\right\} + \left\{{\tau \atop \sigma\mu}\right\}\left\{{\rho \atop \tau\nu}\right\} - \left\{{\rho \atop \tau\mu}\right\}\left\{{\tau \atop \sigma\nu}\right\}, \tag{13}
$$

$$
R_{\mu\nu} = {R^\rho}_{\mu\rho\nu}, \tag{14}
$$

$$
R = g^{\mu\nu}R_{\mu\nu}, \tag{15}
$$

and $\left\{{\lambda \atop \mu\nu}\right\}$ is the Christoffel second kind define by

$$
\left\{{\lambda \atop \mu\nu}\right\} = \frac{1}{2}g^{\sigma\lambda}\left(g_{\mu\sigma,\ \nu}+g_{\nu\sigma,\ \mu}-g_{\mu\nu,\ \sigma}\right), \tag{16}
$$

where , is the differentiation with respect to the coordinate and $T_{\mu\nu}$ is the energy-momentum tensor of a source field of the Lagrangian L_m

$$
T^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta\mathcal{L}_M}{\delta {b^k}_\nu}b^{k\mu} \tag{17}
$$

with $L_M=\mathcal{L}_M/\sqrt{-g}$. The tensors $K_{\mu\nu}$ and $J_{ij\mu}$ are defined as

$$
K_{\mu\nu} = \frac{\kappa}{\lambda}\left(\frac{1}{2}\left[{\epsilon_\mu}^{\rho\sigma\lambda}(T_{\nu\rho\sigma}-T_{\rho\sigma\nu})+{\epsilon_\nu}^{\rho\sigma\lambda}(T_{\mu\rho\sigma}-T_{\rho\sigma\mu})\right]a_\lambda-\frac{3}{2}a_\mu a_\nu-\frac{3}{4}g_{\mu\nu}a^\lambda a_\lambda\right), \tag{18}
$$

$$
J_{ij\mu} = -\frac{3}{2}{b_i}^\rho {b_j}^\sigma \epsilon_{\rho\sigma\mu\nu}a^\nu, \tag{19}
$$

[†]We will denote the symmetric part by ( ), for example, $A_{(\mu\nu)}=(1/2)(A_{\mu\nu}+A_{\nu\mu})$ and the antisymmetric part by the square bracket [ ], $A_{[\mu\nu]}=(1/2)(A_{\mu\nu}-A_{\nu\mu})$.

respectively. The dimensionless parameter λ is defined by

$$\frac{1}{\lambda} = \frac{4}{9}\xi + \frac{1}{3\kappa}. \tag{20}$$

The structure of the Weintzenböck spaces with spherical symmetry and has three unknown functions of radial coordinate is given by Robertson [10] in the form

$$\left(\underset{i}{\lambda}^{\mu}\right) = \begin{pmatrix} iA & iDr & 0 & 0 \\ 0 & B\sin\theta\cos\phi & \frac{B}{r}\cos\theta\cos\phi & -\frac{B\sin\phi}{r\sin\theta} \\ 0 & B\sin\theta\sin\phi & \frac{B}{r}\cos\theta\sin\phi & \frac{B\cos\phi}{r\sin\theta} \\ 0 & B\cos\theta & -\frac{B}{r}\sin\theta & 0 \end{pmatrix}, \tag{21}$$

where the vector $\underset{0}{\lambda}^{\mu}$ has taken to be imaginary in order to preserve the Lorentz signature for the metric, i.e, the functions A and D have to be taken as imaginary. Applying (21) to (10) and (11) the author got

$$\begin{aligned} &when \qquad A = 1, \qquad B = 1 \\ &D = \sqrt{\frac{2m}{r3}\,(1 - e^{-r^3/{r_1}^3})}, \end{aligned} \tag{22}$$

and when $D = 0$

$$\begin{aligned} A &= \frac{1}{\sqrt{1 - \frac{2m}{R}\left(1 - e^{-R^3/{r_1}^3}\right)}}, \\ B &= \sqrt{1 - \frac{2m}{R}\left(1 - e^{-R^3/{r_1}^3}\right)}. \end{aligned} \tag{23}$$

Using (4) the associated metric of the two solutions (22) and (23) is found to be the same as (1) with the stress-energy momentum tensor given by (2). Now we are going to study the singularity problem for both (22) and (23).

3. Singularities

In teleparallel theories we mean by singularity of space-time [14] the singularity of the scalar concomitants of the torsion and curvature tensors.

Using (13), (14), (15), (6) and (8) the scalars of the Riemann-Christofell curvature tensor, Ricci tensor, Ricci scalar, torsion tensor, basic vector, traceless part and the axial vector part of the space-time given by the solution (22) are given by

$$
\begin{aligned}
R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma} &= \frac{12m^2}{{r_1}^{12}r^6}\Bigg[(24{r_1}^6r^6+8{r_1}^9r^3+4{r_1}^{12}+27r^{12})e^{-2r^3/{r_1}^3} \\
&\quad -4(3{r_1}^6r^6+2{r_1}^9r^3+2{r_1}^{12})e^{-r^3/{r_1}^3}+4{r_1}^{12}\Bigg], \\
R^{\mu\nu}R_{\mu\nu} &= \frac{18m^2}{{r_1}^{12}}\left[(8{r_1}^6-12{r_1}^3r^3+9r^6)\right]e^{-2r^3/{r_1}^3} \\
R &= \frac{6m}{{r_1}^6}\left[(3r^3-4{r_1}^3)\right]e^{-r^3/{r_1}^3} \\
T^{\mu\nu\lambda}T_{\mu\nu\lambda} &= \frac{3m}{{r_1}^6r^3\left(e^{-r^3/{r_1}^3}-1\right)}\Bigg[(2{r_1}^3r^3+3{r_1}^6r^3+3r^6)e^{-2r^3/{r_1}^3} \\
&\quad -(2{r_1}^3r^3+6{r_1}^6)e^{-r^3/{r_1}^3}+3{r_1}^6\Bigg], \\
V^{\mu}V_{\mu} &= \frac{9m}{2{r_1}^6r^3\left(e^{-r^3/{r_1}^3}-1\right)}\left[{r_1}^3-({r_1}^3-r^3)e^{-2r^3/{r_1}^3}\right] \\
t^{\mu\nu\lambda}t_{\mu\nu\lambda} &= \frac{9m}{2{r_1}^6r^3\left(1-e^{-r^3/{r_1}^3}\right)}\left[{r_1}^3-({r_1}^3-r^3)e^{-2r^3/{r_1}^3}\right]^2 \\
a^{\mu}a_{\mu} &= 0.
\end{aligned} \tag{24}
$$

The scalars of the Riemann-Christofell curvature tensor, Ricci tensor and Ricci scalar of the solution (23) are the same as given by (24). This is a logic results since both the solutions reproduce the same metric tensor and these scalars mainly depend on the metric tensor. The scalars of torsion tensor, basic vector, traceless part and the axial vector part of the space-time given by the solution (23) are given by

$$
\begin{aligned}
T^{\mu\nu\lambda}T_{\mu\nu\lambda} &= \frac{2}{{r_1}^6r^3\left(r-2m+2me^{-r^3/{r_1}^3}\right)}\Bigg[{r_1}^6\left(2r-3m+3me^{-r^3/{r_1}^3}\right)^2 \\
&\quad -4\sqrt{r}{r_1}^6\left(r-2m+2me^{-r^3/{r_1}^3}\right)^{3/2}+3m^2r^3e^{-r^3/{r_1}^3}\left(3r^3e^{-r^3/{r_1}^3}-2{r_1}^3+2{r_1}^3e^{-r^3/{r_1}^3}\right)\Bigg] \\
V^{\mu}V_{\mu} &= \frac{1}{{r_1}^6r^4\left(r-2m+2me^{-r^3/{r_1}^3}\right)^2}\Bigg[2r{r_1}^3\left(r-2m+2me^{-r^3/{r_1}^3}\right) \\
&\quad +\sqrt{r2-2mr+2mre^{-r^3/{r_1}^3}}\left\{3{r_1}^3m-3{r_1}^3me^{-r^3/{r_1}^3}-2r{r_1}^3+3mr^3e^{-r^3/{r_1}^3}\right\}\Bigg]^2, \\
t^{\mu\nu\lambda}t_{\mu\nu\lambda} &= \frac{1}{{r_1}^6r^4\left(r-2m+2me^{-r^3/{r_1}^3}\right)^2}\Bigg[r{r_1}^3\left(r-2m+2me^{-r^3/{r_1}^3}\right) \\
&\quad +\sqrt{r2-2mr+2mre^{-r^3/{r_1}^3}}\left\{3{r_1}^3m-3{r_1}^3me^{-r^3/{r_1}^3}-r{r_1}^3-3mr^3e^{-r^3/{r_1}^3}\right\}\Bigg]^2.
\end{aligned} \tag{25}
$$

As is clear from (24) and (25) that the scalars of the torsion, basic vector and the traceless part of the two solutions (22) and (23) are quite different in spite that they gave the same associated metric (1). Also as we see from (24) and (25) that as $r \to 0$ all the scalars take finite value.

4. The Stability condition

In the background of gravitational field the trajectories are represented by the geodesic equation

$$\frac{d^2x^\lambda}{ds^2} + \left\{ {}^{\lambda}_{\mu\nu} \right\} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \tag{26}$$

where $\frac{dx^\mu}{ds}$ is the velocity four vector, s is a parameter varying along the geodesic. It is well know that the perturbation of the geodesic will lead to deviation [1]

$$\frac{d^2\zeta^\lambda}{ds^2} + 2\left\{ {}^{\lambda}_{\mu\nu} \right\} \frac{dx^\mu}{ds}\frac{d\zeta^\nu}{ds} + \left\{ {}^{\lambda}_{\mu\nu} \right\}_{,\ \rho} \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\zeta^\rho = 0, \tag{27}$$

where ζ^ρ is the deviation 4-vector.

Applying (26), (27) in (1) we get for the geodesic equations

$$\frac{d^2t}{ds^2} = 0, \qquad \frac{1}{2}\eta'(r)\left(\frac{dt}{ds}\right)^2 - r\left(\frac{d\phi}{ds}\right)^2 = 0, \qquad \frac{d^2\theta}{ds^2} = 0, \qquad \frac{d^2\phi}{ds^2} = 0, \tag{28}$$

and for the geodesic deviation

$$\begin{aligned}
&\frac{d^2\zeta^0}{ds^2} + \frac{\eta'(r)}{\eta(r)}\frac{dt}{ds}\frac{d\zeta^1}{ds} = 0 \\
&\frac{d^2\zeta^1}{ds^2} + \eta(r)\eta'(r)\frac{dt}{ds}\frac{d\zeta^0}{ds} - 2r\eta(r)\frac{d\phi}{ds}\frac{d\zeta^3}{ds} \\
&+ \left[\frac{1}{2}\left(\eta'^2(r) + \eta(r)\eta''(r)\right)\left(\frac{dt}{ds}\right)^2 - \left(\eta(r) + r\eta'(r)\right)\left(\frac{d\phi}{ds}\right)^2\right]\zeta^1 = 0, \\
&\frac{d^2\zeta^2}{ds^2} + \left(\frac{d\phi}{ds}\right)^2\zeta^2 = 0, \\
&\frac{d^2\zeta^3}{ds^2} + \frac{2}{r}\frac{d\phi}{ds}\frac{d\zeta^1}{ds} = 0,
\end{aligned} \tag{29}$$

where $\eta(r) = (1 - \frac{R_g(r)}{r})$, $\eta'(r) = \frac{d\eta(r)}{dr}$ and we have consider the circular orbit in the plane

$$\theta = \frac{\pi}{2}, \qquad \frac{d\theta}{ds} = 0, \qquad \frac{dr}{ds} = 0. \tag{30}$$

Using (30) in (1) we get

$$\eta(r)\left(\frac{dt}{ds}\right)^2 - r^2\left(\frac{d\phi}{ds}\right)^2 = 1, \tag{31}$$

from (28) and (31) we obtain

$$\left(\frac{d\phi}{ds}\right)^2 = \frac{\eta'(r)}{r(2\eta(r) - r\eta'(r))}, \qquad \left(\frac{dt}{ds}\right)^2 = \frac{2}{2\eta(r) - r\eta'(r)}. \tag{32}$$

The variable s in (29) can be eliminated and we can rewrite it in the form

$$
\begin{aligned}
&\frac{d^2\zeta^0}{d\phi^2}+\frac{\eta'(r)}{\eta(r)}\frac{dt}{d\phi}\frac{d\zeta^1}{d\phi}=0\\
&\frac{d^2\zeta^1}{d\phi^2}+\eta(r)\eta'(r)\frac{dt}{d\phi}\frac{d\zeta^0}{d\phi}-2r\eta(r)\frac{d\zeta^3}{d\phi}\\
&+\left[\frac{1}{2}\left(\eta'^2(r)+\eta(r)\eta''(r)\right)\left(\frac{dt}{d\phi}\right)^2-(\eta(r)+r\eta'(r))\right]\zeta^1=0,\\
&\frac{d^2\zeta^2}{d\phi^2}+\zeta^2=0,\\
&\frac{d^2\zeta^3}{ds^2}+\frac{2}{r}\frac{d\zeta^1}{d\phi}=0.
\end{aligned}
\tag{33}
$$

As is clear from the third equations of (33) that it represent a simple harmonic motion, this means that the motion in the plan $\theta=\pi/2$ is stable.

Assuming now the solution of the remaining equations given by

$$
\zeta^0=A_1e^{i\omega\phi},\qquad \zeta^1=A_2e^{i\omega\phi},\qquad \zeta^3=A_3e^{i\omega\phi},
\tag{34}
$$

where A_1, A_2, A_3 are constants to be determined. From (34) and (33) we get

$$
\begin{aligned}
&\frac{1}{rr_1{}^3\left(r_1{}^3-(r_1{}^3+3r^3)e^{-r^3/r_1{}^3}\right)}\Big[r_1{}^6(r-6m)+(12mr_1{}^6-rr_1{}^6-9r^4r_1{}^3+42mr^3r_1{}^3\\
&+9r^7-18mr^6)e^{-r^3/r_1{}^3}-(6mr_1{}^6+42mr^3r_1{}^3+18mr^6)e^{-r^3/r_1{}^3}\Big]>0,
\end{aligned}
\tag{35}
$$

which is the condition of the stability for the vacuum non singular black hole solution.

5. Main results

The main results can be summarized as follows

1) The singularity problem for the two solutions (22) and (23) obtained before [9] has been studied.

i) As we see from (24) that all the scalars have a finite value as $r\to 0$ given by $96\frac{m^2}{r_1{}^6}$, $144\frac{m^2}{r_1{}^6}$, $-24\frac{m}{r_1{}^3}$, $-12\frac{m}{r_1{}^3}$, $-18\frac{m}{r_1{}^3}$, 0 and 0 respectively, i.e., Remain finite and tend to the de Sitter value for the scalars of the Riemann Christoffel tensor, Ricci tensor and Ricci scalar.

ii) For large r the scalars of (24) have the value $48\frac{m^2}{r^6}$, 0, 0, $-9\frac{m}{r^3}$, $-\frac{9}{2}\frac{m}{r^3}$, $-\frac{9}{2}\frac{m}{r^3}$ and 0 respectively which are the Schwarzschild value for the Riemann Christoffel tensor, Ricci tensor and Ricci scalar.

iii) For the solution (25) we found that all the scalars have a finite value as $r\to 0$ given by $96\frac{m^2}{{r_1}^6}$, $144\frac{m^2}{{r_1}^6}$, $-24\frac{m}{{r_1}^3}$, 0, 0, 0 and 0 respectively, i.e., Remain finite and tend to the de Sitter value for the Riemann Christoffel tensor, Ricci tensor and Ricci scalar same as of the solution (24). The values of the scalars of the torsion, basic vector and traceless part are different for the solutions (24) and (25) when $r \rightarrow 0$. This may be due to the fact that the asymptotic behavior of the parallel vector fields of the two solutions (24) and (25) is quite different.

iv) For large r the scalars of (24) have the value $48\frac{m^2}{r^6}$, 0, 0, $\frac{2(4r^2-12rm-4r\sqrt{r^2-2mr}+8m\sqrt{r^2-2mr}+9m^2)}{r^3(r-2m)}$, $\frac{(2r^2-4rm-2r\sqrt{r^2-2mr}+3m\sqrt{r^2-2mr})^2}{r^4(r-2m)^2}$, $\frac{(r^2-2rm-r\sqrt{r^2-2mr}+3m\sqrt{r^2-2mr})^2}{r^4(r-2m)^2}$ and 0 respectively which are the Schwarzschild value for the Riemann Christoffel tensor, Ricci tensor and Ricci scalar same as of the solution (24). The values of the scalars of the torsion, basic vector traceless are different for the solutions (24) and (25) for large r due to the reason given above.

2) The stability condition for the metric of the vacuum non singular black hole is derived (35). From this condition we can see that.

i) As $r \rightarrow 0$ the value of (35) is finite.

ii) As r becomes large the value of (35) takes the value $r > 6m$ which the is condition of the stability of the Schwarzschild solution.

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